AIAA JOURNAL
Vol. 42, No. 5, May 2004

Volume Averaged Pressure Interactions for Dispersed Droplet Phase Modeling of Multiphase Flow

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This study addresses cross-phase interactions and momentum exchange between dispersed (droplet) and carrier (air) phases of multiphase flow. Alternative formulations are developed for the volume averaged interfacial pressure and apparent mass forces. Algebraic source terms in the momentum equation can accommodate the interfacial shear stresses. Multiphase turbulent stresses in the carrier phase are modeled with a Boussinesq assumption. New interfacial pressure modeling in an Eulerian formulation with volume averaging is developed.

Nomenclature

a_i	=	surface of the phase interface
a_{kw}	=	surface of phase k in contact with the wall surface
C_D	=	drag coefficient
D_d	=	droplet diameter
$\underline{g}$	=	gravity
$\bar{\bar{I}}$	=	identity tensor
$\boldsymbol{j}_k$	=	flux of the general scalar quantity
$\dot{m}_k$	=	mass flux for the particular phase
$\boldsymbol{n}_k, \boldsymbol{n}_1, \boldsymbol{n}_2$	=	normal vector for the particular phase
$\boldsymbol{n}_{kw}$	=	normal vector for the particular phase at the wall surface
p	=	pressure
p_k	=	local pressure of phase k
$\langle p_k \rangle$	=	volume-averaged pressure of phase k
$\langle p_{\mathrm{ki}} \rangle$	=	volume-averaged interfacial pressure of phase k
Re_r	=	relative (droplet-air) Reynolds number
$\underline{S}$	=	surface
$\hat{S}$	=	source term
V	=	volume
$\boldsymbol{v}$	=	phase velocity [$\boldsymbol{v}$, dispersed phase; $\boldsymbol{v}_a$, carrier (air) phase]
β	=	volume fraction
ρ	=	density
$\bar{\bar{\tau}}$	=	shear-stress tensor
ψ	=	general scalar quantity
∇	=	gradient operator

Subscripts

d	=	droplet
i	=	interfacial
k	=	particular phase
r	=	relative
x, y	=	coordinate directions

Received 5 September 2002; revision received 20 August 2003; accepted for publication 21 January 2004.

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I. Introduction

ACCURATE prediction of multiphase flows is essential for solving technological problems in various engineering applications. For example, multiphase flows with droplets contribute to inflight icing of helicopter surfaces exposed to the impinging droplets. These incoming droplets largely affect the heat balance on the iced aircraft surface,[1,2] as well as the final ice shape.[3] In many cases, averaged physical quantities are sufficient for design purposes. Previous studies have adopted Eulerian volume-averaging techniques (denoted by $\langle \rangle$) for various types of multiphase flows (i.e., Refs. 4–6). However, their extensions to multiphase flows with droplets often involve modeling approximations without a detailed documented basis in the volume averaging. One particular example of interest in this paper is the averaged cross-phase pressure interactions. In this paper, a two-fluid multiphase model will be considered in this regard.

The role of the pressure interactions in multiphase modeling and their physical significance have been studied previously, that is, Sha and Soo,[7] Boure,[8] Prosperetti and Jones,[9] Marchioro et al.,[10] and others. Various aspects of the representation of the pressure force term (denoted by $[P\nabla\beta]$) in the phase momentum equations have been documented in the literature. Ishii[11] cancels out this term via the interfacial source term when the bulk mean pressure is nearly equal to the interfacial mean pressure. Gidaspow and Solbring[12] pointed out limitations with this partial pressure model, as it can yield unrealistically high pressures. However, Soo[13] suggested that there is no need to drop the $[P\nabla\beta]$ term for purposes of solution stabilization. Gidaspow and Solbring[12] called this pressure term an "extra type diffusive force" and outlined that the pressure in this expression is not simply the thermodynamic pressure. Jackson[14] and Medlin et al.[15] called the term an "effective force without giving an exact value," Gidaspow[16] suggested that $[P\nabla\beta]$ might be dropped for the case of stratified flow but not in suspensions. Furthermore, Sha and Soo[7] describe the significance of the $[P\nabla\beta]$ term for adiabatic compression of a gas bubble, involving nearly zero velocities of both phases (liquid and gas). Their analysis entailed the kinetic energy and continuity equations with thermodynamic relations for a perfect gas.

A form of the phasic momentum equations for two-phase flow is discussed by Boure.[8] Volume-averaged phase momentum equations are used, and a treatment of the bulk pressure term is shown. The pressure deviations at the phase interface are not fully documented. Prosperetti and Jones[9] have investigated the volume-averaged dispersed and continuous pressure forces in a detailed manner. Although the pressure forces in the continuous phase affect the dispersed-phase motion, the pressure within the dispersed phase does not appreciably contribute to fluid motion in the continuous phase under certain conditions.

Marchioro et al.[10] have used ensemble averaging for modeling a mixture pressure and viscous stresses in a dispersed two-phase flow. A pressure force is modeled from the bulk mixture total stress of all phases in contact. Nigmatulin[17] has used mixture modeling to address such pressure forces. Park et al.[18] used an ensemble-averaging technique for the pressure term in a two-fluid multiphase model, but the difference between the averaged pressure in the bulk flow and the interfacial pressure was not fully documented. Also, assumptions of inviscid and irrotational flow were made. Joseph and Lundgren[19] have used ensemble averaging to derive the relevant transport equations, while the pressure field was assumed to be uniform.

Turbulent fluctuations in single-phase flows, represented by single-phase Reynolds stresses, appear as a result of the fluctuating velocity field. These Reynolds stresses can be modeled by a Boussinesq gradient assumption. This assumption relates velocity fluctuations to the mean velocities.[20] In multiphase modeling, averaging techniques are used to describe filtered physical fields, free of deviations.[21] These deviations represent information lost in the averaging procedure. Many physical reasons exist for these deviations. If the turbulence is the most important source, a deviating velocity field is modeled through the multiphase Reynolds stresses. Modeling of multiphase Reynolds stresses is more complicated than modeling of single-phase Reynolds stresses.[18,21,22] For a dilute disperse flow, a Boussinesq assumption is used to relate a deviating velocity field to an averaged velocity field.

In this paper, a rigorously formulated Eulerian model of multiphase flow with droplets is developed, particularly for applications involving helicopter icing. A detailed derivation of the volume-averaged mass, momentum and energy equations for the dispersed (droplet) phase is documented, including a new derivation and treatment of the pressure terms. Also, unlike other conventional droplet flow models that track individual droplet trajectories, the current approach can potentially reduce the computational time by applying spatial averaging of the governing equations.[3] Various previous types of averaging procedures have been documented in the technical literature, such as time averaging, spatial averaging, ensemble averaging, or combinations of these procedures. In previous studies involving multiphase flows with droplets,[23,24] a final reduced form of the volume-averaged momentum equations in the dispersed phase has been presented. However, various modeling assumptions and approximations are adopted beforehand. In the averaging procedures, certain microscopic information is lost through the averaging integration process, especially at the phase interface(s). Information lost through spatial averaging can be supplied back through other relations, such as constitutive or macroscopic/microscopic relationships. Spatial averaging is most effective when the integrated quantities do not change significantly over the selected averaging control volume.

A detailed volume-averaging procedure for the dispersed phase is developed in this paper, based on extensions of previous work by Banerjee and Chan,[4] Naterer,[3,25] and Crowe.[21] This particularly involves the cross-phase pressure interactions, which can have an important role in the dynamics of the dispersed phase.[22] The bulk phase pressure and an interfacial pressure are modeled in a two-fluid formulation. The local and volume-averaged pressure forces contribute to the motion of both phases in contact. The pressure field in the dispersed phase can affect the continuous phase, particularly if the dispersed phase is moved suddenly from a region to another region with different bulk pressures. Detailed treatment is considered for the pressure effects at the interface between dispersed and continuous phases. Unlike previous studies (i.e., Prosperetti and Jones[9]), interfacial pressure drag and the apparent mass force per unit volume are modeled in a more detailed manner. For example, a pressure term in the momentum equations $[P\nabla\beta]$ is reformulated, while shedding new light on the detailed significance of the interfacial pressure drag and apparent mass force terms.

The newly modeled terms, such as interfacial pressure drag and apparent mass, have been applied to a droplet/air two-fluid model. This approach differs from previous formulations (i.e., Prosperetti and Jones[9]). Afterwards, an interfacial pressure drag force per unit volume and apparent mass force per unit volume were assumed to coincide. They have been grouped under one interfacial surface integral, thereby avoiding certain modeling difficulties associated with the term $[P\nabla\beta]$ in the momentum equations. After they were grouped together with the shear interfacial and shear wall terms, all of these terms have been further modeled by an algebraic term. Pressure deviations at the phase interface are considered, as well as the role of pressure terms in the energy equation.[26]

In this paper, multiphase Reynolds stresses arise as a result of the velocity deviations. In a dilute disperse flow, the turbulence in the carrier phase is mainly responsible for these velocity deviations.[23] Any information lost in the averaging procedure will be supplied back through the Boussinesq assumption. This assumption relates deviating velocities to the gradients of the volume-averaged velocities of the disperse phase. A Boussinesq assumption has certain limitations. With this assumption, detailed droplet behavior as a result of the presence of a wall cannot be fully described.

II. Multiphase Flow Formulation

The governing equations of multiphase flow and heat transfer can be obtained by performing conservation balances over individual phases within a multiphase averaging control volume.[25] A sample representation of a multiphase averaging control volume with droplets is shown in Fig. 1. In this paper, the general transport equation is volume averaged, based on the Leibnitz and Gauss theorems for both phases in Cartesian coordinates.

A. General Scalar Transport Equation

The general transport equation for phase k is given by[9]

$$\frac{\partial(\rho_k\psi_k)}{\partial t} + \nabla\cdot(\rho_k\psi_k\boldsymbol{v}_k + \boldsymbol{j}_k) - \rho_k\hat{S}_k = 0 \tag{1}$$

From the Leibnitz and the Gauss theorems (using volume averaging), respectively, we have for each phase

$$\int_{V_k}\frac{\partial\rho_k\psi_k}{\partial t}\,\mathrm{d}V = \frac{\partial}{\partial t}\int_{V_k}\rho_k\psi_k\,\mathrm{d}V - \int_{a_i}\rho_k\psi_k(\boldsymbol{v}_i\cdot\boldsymbol{n}_k)\,\mathrm{d}S \tag{2}$$

$$\int_{V_k}\nabla\cdot(\rho_k\psi_k\boldsymbol{v}_k + \boldsymbol{j}_k)\,\mathrm{d}V = \frac{\partial}{\partial x}\int_{V_k}\boldsymbol{n}_x\cdot(\rho_k\psi_k\boldsymbol{v}_k + \boldsymbol{j}_k)\,\mathrm{d}V + \frac{\partial}{\partial y}\int_{V_k}\boldsymbol{n}_y\cdot(\rho_k\psi_k\boldsymbol{v}_k + \boldsymbol{j}_k)\,\mathrm{d}V + \int_{a_i+a_{kw}}\boldsymbol{n}_k\cdot(\rho_k\psi_k\boldsymbol{v}_k + \boldsymbol{j}_k)\,\mathrm{d}S \tag{3}$$

The variables $\rho_k\psi_k$, $\boldsymbol{j}_k$, and $\hat{S}_k$ refer to the conserved quantity in the kth phase, the flux of ψ_k, and the source of ψ_k. Equation (2) is written for a specific phase occupied by the portion of the averaging control volume containing that phase, namely, V_k, rather than the mixture occupying the full averaging control volume V (Fig. 1).

Integrating Eq. (1) over the volume of phase k, V_k, and substituting Eqs. (2) and (3) into Eq. (1) yields the following integral equation

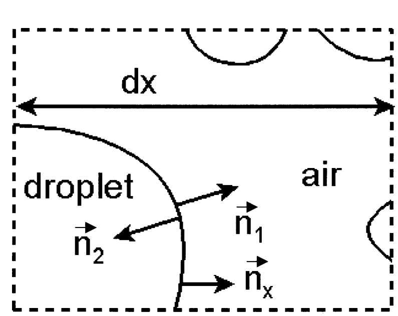

Fig. 1 Schematic of multiphase averaging control volume.

for the conserved quantity $\rho_k \psi_k$:

$$\frac{\partial}{\partial t}\int_{V_k}\rho_k\psi_k\,\mathrm{d}V+\frac{\partial}{\partial x}\int_{V_k}\boldsymbol{n}_x\cdot(\rho_k\psi_k\boldsymbol{v}_k+\boldsymbol{j}_k)\,\mathrm{d}V+\frac{\partial}{\partial y}\int_{V_k}\boldsymbol{n}_y\cdot(\rho_k\psi_k\boldsymbol{v}_k+\boldsymbol{j}_k)\,\mathrm{d}V-\int_{V_k}\rho_k\hat{S}_k\,\mathrm{d}V=\int_{a_i}\rho_k\psi_k(\boldsymbol{v}_i\cdot\boldsymbol{n}_k-\boldsymbol{n}_k\cdot\boldsymbol{v}_k)\,\mathrm{d}S-\int_{a_i+a_{kw}}(\boldsymbol{n}_k\cdot\boldsymbol{j}_k+\boldsymbol{n}_{kw}\cdot\boldsymbol{j}_k)\,\mathrm{d}S \tag{4}$$

This equation outlines the transport processes at the interfacial and wall surfaces. In the first term on the right-hand side, the cross-phase mass transfer can be recognized as

$$-\dot{m}_k=\rho_k\boldsymbol{n}_k(\boldsymbol{v}_k-\boldsymbol{v}_i) \tag{5}$$

The volume-averaged scalar equation for the conserved quantity becomes

$$\frac{\partial}{\partial t}\beta_k\langle\rho_k\psi_k\rangle+\frac{\partial}{\partial x}\beta_k\langle\boldsymbol{n}_x\cdot(\rho_k\psi_k\boldsymbol{v}_k+\boldsymbol{j}_k)\rangle+\frac{\partial}{\partial y}\beta_k\langle\boldsymbol{n}_y\cdot(\rho_k\psi_k\boldsymbol{v}_k+\boldsymbol{j}_k)\rangle-\beta_k\langle\rho_k\hat{S}_k\rangle=\frac{1}{V}\int_{a_i}(\dot{m}_k\psi_k+\boldsymbol{j}_k\cdot\boldsymbol{n}_k)\,\mathrm{d}S-\frac{1}{V}\int_{a_{kw}}\boldsymbol{n}_{kw}\cdot\boldsymbol{j}_k\,\mathrm{d}S \tag{6}$$

where volume averages of the integrals of a conserved function, that is, $b_k=\rho_k\psi_k$, are defined by

$$\langle b_k\rangle=\frac{1}{V_k}\int_{V_k}b_k\,\mathrm{d}V,\qquad\langle b_k\rangle_i=\frac{1}{V}\int_{a_i}b_k\,\mathrm{d}S \tag{7}$$

The parameter $\beta_k=V_k/V$ denotes the phase volume fraction.

B. Phase Fraction (Mass) Equation

Substituting $\psi_k=1$, $\boldsymbol{j}_k=0$, and $\hat{S}_k=0$ in Eq. (4), the integral equation for the conserved quantity ρ_k becomes

$$\frac{\partial}{\partial t}\int_{V_k}\rho_k\,\mathrm{d}V-\int_{a_i}\rho_k(\boldsymbol{v}_i\cdot\boldsymbol{n}_k)\,\mathrm{d}S+\frac{\partial}{\partial x}\int_{V_k}\boldsymbol{n}_x\cdot(\rho_k\boldsymbol{v}_k)\,\mathrm{d}V+\frac{\partial}{\partial y}\int_{V_k}\boldsymbol{n}_y\cdot(\rho_k\boldsymbol{v}_k)\,\mathrm{d}V+\int_{a_i}\boldsymbol{n}_k\cdot(\rho_k\boldsymbol{v}_k)\,\mathrm{d}S=0 \tag{8}$$

The interfacial term can be expressed as

$$\int_{a_i}[\rho_k(\boldsymbol{v}_i\cdot\boldsymbol{n}_k)-\boldsymbol{n}_k\cdot(\rho_k\boldsymbol{v}_k)]\,\mathrm{d}S=-\int_{a_i}[\rho_k\boldsymbol{n}_k\cdot(\boldsymbol{v}_k-\boldsymbol{v}_i)]\,\mathrm{d}S=-\int_{a_i}\dot{m}_k\,\mathrm{d}S \tag{9}$$

Applying the dot products with the unit normal vectors and volume averaging, Eq. (8) becomes

$$\frac{\partial}{\partial t}(\beta_d\langle\rho_k\rangle)+\frac{\partial}{\partial x}(\beta_d\langle\rho_k v_{kx}\rangle)+\frac{\partial}{\partial y}(\beta_d\langle\rho_k v_{ky}\rangle)=-\langle\dot{m}_k\rangle_i \tag{10}$$

The term on the right-hand side of this equation represents the interphase mass transfer as a result of the mass transfer of the droplet phase across the phase interface. It leads to a momentum change, and therefore a corresponding term appears in the momentum equation. In the absence of droplet evaporation, coalescence, or collisions, this term becomes negligible.

C. Momentum Equations (with Pressure Treatment)

The droplet momentum equations are determined after setting $\psi_k=\boldsymbol{v}_k,\boldsymbol{j}_k=p_k\bar{\bar{I}}-\bar{\bar{\tau}}_k$, and $\hat{S}_k=\boldsymbol{F}_k$ in Eq. (6). After inserting these relations and taking the dot product with $\boldsymbol{n}_x$, and then with $\boldsymbol{n}_y$, we obtain two separate momentum equations in the x [Eqs. (19) and (21)] and in the y [Eq. (22)] directions. Their distinctive derivation procedure and final form with respect to the one-dimensional multiphase equations, as commonly found in the literature, can be readily extended to three dimensions by including analogous terms in the z direction.

The hydrodynamic forces, such as drag forces (i.e., Stokes, Faxen, and Basset), apparent mass force, and lift forces (i.e., Magnus and Saffman)[21] are interfacial pressure and interfacial shear forces appearing through the interphase terms in the momentum equations for the specific phase, not externally acting or explicitly included forces. These terms represent interfacial interactions between the phases in contact (including wall surfaces). These terms are active terms in the momentum equations for the particular phase. When these forces are evaluated for the specific situation and incorporated into the momentum equations, they can be grouped together with externally acting and explicitly added body volumetric forces (i.e., Corriolis, Coloumb, and London) into the source term on the right-hand side of the momentum equations. This can be useful for purposes of numerical modeling. Regarding interfacial forces, the Basset and apparent mass forces are unsteady-state interfacial forces. The local difference between the velocities of the phases in contact is not constant. It is changing with time. This local acceleration, expressed through these interfacial forces, changes the momentum of the disperse phase at its interface.

In unsteady-state motion of the disperse phase, both steady-state and unsteady-state interfacial forces can influence the motion. In this case, unsteady-state interfacial forces can be introduced on the left-hand side of the momentum equations of the disperse phase, while the rest of the interfacial forces can be grouped with the externally applied body forces in the source term on the right-hand side of the momentum equations. This can provide useful linearization for a numerical formulation, and it allows the equations to be discretized and written in terms of a single active variable, so that the standard algebraic solvers can be used.

In steady-state motion of the disperse phase, in the presence of steady-state interfacial forces, all interfacial forces can be grouped together with externally applied body forces. These forces can then be modeled as a source term.

Volume averaging is performed for wavy flows. A velocity field is decomposed into the volume-averaged component, denoted by $\langle v\rangle$, and a deviating part of the volume-averaged component, denoted as δv. In turbulent flow modeling, a deviating velocity component δv will be related to the volume-averaged velocity $\langle v\rangle$ through the Boussinesq gradient assumption. The second (convective) term in Eq. (6), for example, becomes

$$\boldsymbol{n}_x\cdot\frac{\partial}{\partial x}\langle\boldsymbol{n}_x\cdot(\rho_k\psi_k\boldsymbol{v}_k+\boldsymbol{j}_k)\rangle=\frac{\partial}{\partial x}[\rho_k(\langle v_{kx}\rangle\langle v_{kx}\rangle+\langle\delta v_{kx}\delta v_{kx}\rangle)+\langle p_k-\tau_{k,xx}\rangle] \tag{11}$$

Now, consider the pressure part of the fifth term in Eq. (6), that is,

$$-\frac{1}{V}\int_{a_i}(\dot{m}_k\psi_k+\boldsymbol{j}_k\cdot\boldsymbol{n}_k)\,\mathrm{d}S=-\frac{1}{V}\int_{a_i}[\dot{m}_k\boldsymbol{v}_k+\boldsymbol{n}_k\cdot(p_k\bar{\bar{I}}-\bar{\bar{\tau}}_k)]\,\mathrm{d}S$$

Volume averaging of these terms has not been fully documented in the literature, partly because of the difficulties encountered when averaging is performed over a dispersed (rather than continuous) phase. This paper attempts to shed light on such pressure modeling arising from the interactions between the carrier (air) and dispersed (droplet) phases.

Modeling of the fifth term of Eq. (6) outlines the importance of pressure mechanisms at the phase interface. Taking the dot product of the pressure part in the fifth term of Eq. (6) with $\boldsymbol{n}_x$ yields the following result:

$$-\frac{1}{V}\int_{a_i}\boldsymbol{n}_k\cdot(\boldsymbol{n}_x p_k)\,\mathrm{d}S = (\langle p_k\rangle+\Delta p_{\mathrm{ki}})\frac{\partial\beta_k}{\partial x}-\frac{1}{V}\int_{a_i}\boldsymbol{n}_k\cdot(\boldsymbol{n}_x\Delta p'_{\mathrm{ki}})\,\mathrm{d}S \tag{12}$$

An assembly procedure of the pressure parts in Eq. (12) is needed. It is outlined hereafter. The pressure under the integral on the left-hand side of the Eq. (12) is modeled as $p_k = \langle p_k\rangle + \Delta p'_{\mathrm{ki}} + \Delta p_{\mathrm{ki}}$ in order to fully account for the pressure effects at the interface between phases in contact. The term $\Delta p_{\mathrm{ki}} = \langle p_{\mathrm{ki}}\rangle - \langle p_k\rangle$ accounts for the difference between the volume-averaged interfacial and volume-averaged phase pressures. The term $\Delta p'_{\mathrm{ki}} = p_k - \langle p_{\mathrm{ki}}\rangle$ accounts for the difference between the local and volume-averaged interfacial pressures. These pressure terms can be further modeled and accommodated in the momentum equations, but that modeling is beyond the scope of this paper. The difference $\Delta p'_{\mathrm{ki}}$ represents an additional integral term on the right-hand side of the momentum equation (12). In this paper, the term $\Delta p'_{\mathrm{ki}}$ is not modeled, and it is left uncombined with the other terms in the final momentum equations. After the remaining parts, such as $\langle p_k\rangle$ and Δp_{ki} of the pressure under the integral on the left-hand side of Eq. (12) are rearranged, the following expression is obtained:

$$\frac{1}{V}\int_{a_i}\boldsymbol{n}_k\cdot[\boldsymbol{n}_x(\langle p_k\rangle+\Delta p_{\mathrm{ki}})]\,\mathrm{d}S = \frac{1}{V}\int_{V_k}\boldsymbol{\nabla}\cdot[\boldsymbol{n}_x(\langle p_k\rangle+\Delta p_{\mathrm{ki}})]\,\mathrm{d}V$$
$$-\frac{1}{V}\frac{\partial}{\partial x}\int_{V_k}\boldsymbol{n}_x\cdot[\boldsymbol{n}_x(\langle p_k\rangle+\Delta p_{\mathrm{ki}})]\,\mathrm{d}V$$
$$-\frac{1}{V}\frac{\partial}{\partial y}\int_{V_k}\boldsymbol{n}_y\cdot[\boldsymbol{n}_x(\langle p_k\rangle+\Delta p_{\mathrm{ki}})]\,\mathrm{d}V \tag{13}$$

Performing the necessary vector operations yields the following expression for the interfacial pressure on the right-hand side of the x-momentum equation:

$$-\frac{1}{V}\int_{a_i}\boldsymbol{n}_k\cdot(\boldsymbol{n}_x p_k)\,\mathrm{d}S = \frac{\partial}{\partial x}\langle(\langle p_k\rangle+\Delta p_{\mathrm{ki}})\rangle\beta_k$$
$$-\beta_k\frac{\partial}{\partial x}(\langle p_k\rangle+\Delta p_{\mathrm{ki}}) \tag{14}$$

A deviating component of the pressure is considered here to fully account for the pressure interactions at the interfacial surface. The deviation $\Delta p'_{\mathrm{ki}}$ appears when the term Δp_{ki} is not uniform over the interfacial area. Such differences could arise from velocity nonuniformities around a droplet. In a dilute dispersed flow, a turbulent carrier phase is a source of pressure deviations at the interface of the disperse flow (droplets, bubbles, etc.). The interfacial pressure differences are distinct from spatially averaged values in the sixth term of Eq. (6), which involves their interactions with the physical boundary (wall). Equation (14) is customized in the final form, expressed by the first term on the right-hand side of Eq. (12).

Expressing and assembling the fifth term of the momentum equation (6) in terms of the volume-averaged quantities yields

$$-\frac{1}{V}\int_{a_i}\boldsymbol{n}_x\cdot[\dot{m}_k\boldsymbol{v}_k+\boldsymbol{n}_k(p_k\bar{\bar{I}}-\bar{\bar{\tau}}_k)]\,\mathrm{d}S$$
$$= -\langle\dot{m}_k v_{kx}\rangle_i+(\langle p_k\rangle+\Delta p_{\mathrm{ki}})\frac{\partial\beta_k}{\partial x}-\langle\Delta p'_{\mathrm{ki}}\rangle_i+\langle\boldsymbol{n}_k\bar{\tau}_{kx}\rangle_i \tag{15}$$

This term is already written as a part of the right-hand side of the dispersed phase momentum equation in the x direction. In Eq. (15), a constant surface tension along the interfacial boundary is assumed. When the phase composition or relative velocity between phases changes, an apparent force is created. The third term on the right-hand side of Eq. (15), $\langle\Delta p'_{\mathrm{ki}}\rangle_i$, leads to this apparent mass force, which arises to accelerate the mass of the surrounding continuous phase in the immediate vicinity of the dispersed phase. This term is a force per unit volume in the momentum equation, and it affects the change of momentum for both phases.

In Eq. (15), certain waviness between the dispersed and continuous phases can exist along the interfacial boundary, so that an apparent mass effect can arise. However, the large density difference between phases for water droplets in air diminishes this apparent mass effect. If the pressure is assumed to be constant and uniform within both phases in the averaging control volume, then both deviating pressure components become negligible. This means that the pressure terms $\beta_k(\partial\langle p_k\rangle/\partial x)$ and $\beta_k(\partial\langle p_k\rangle/\partial y)$ become valid only within the carrier phase, while the physical effects at the surface of the droplet are lost.

The pressure part of the interfacial surface equation (15) is written as

$$-\frac{1}{V}\int_{a_i}\boldsymbol{n}_x\cdot[\boldsymbol{n}_k(p_k)]\,\mathrm{d}S = \langle p_k\rangle\frac{\partial\beta_k}{\partial x}+\Delta p_{\mathrm{ki}}\frac{\partial\beta_k}{\partial x}$$
$$-\int_{a_i}\boldsymbol{n}_x\cdot[\boldsymbol{n}_k(p_k-\langle p_{\mathrm{ki}}\rangle)]\,\mathrm{d}S \tag{16}$$

By grouping terms together with respect to x and y directions, the linear momentum equations for the volume-averaged quantities can be obtained. Using

$$\frac{\partial}{\partial x}\beta_k\langle p_k\rangle = \beta_k\frac{\partial\langle p_k\rangle}{\partial x}+\langle p_k\rangle\frac{\partial\beta_k}{\partial x} \tag{17}$$

$$\frac{\partial}{\partial y}\beta_k\langle p_k\rangle = \beta_k\frac{\partial\langle p_k\rangle}{\partial y}+\langle p_k\rangle\frac{\partial\beta_k}{\partial y} \tag{18}$$

the x-momentum equation becomes

$$\frac{\partial}{\partial t}(\beta_k\rho_k\langle v_{kx}\rangle)+\frac{\partial}{\partial x}(\beta_k\rho_k\langle v_{kx}\rangle\langle v_{kx}\rangle)+\frac{\partial}{\partial y}(\beta_k\rho_k\langle v_{kx}\rangle\langle v_{ky}\rangle)$$
$$+\beta_k\frac{\partial}{\partial x}\langle p_k\rangle-\Delta p_{\mathrm{ki}}\frac{\partial\beta_k}{\partial x}-\frac{\partial}{\partial x}(\beta_k\langle\tau_{kxx}\rangle)-\frac{\partial}{\partial y}(\beta_k\langle\tau_{kyx}\rangle)$$
$$-\beta_k\rho_k\langle F_{kx}\rangle = -\langle\dot{m}_k v_{kx}\rangle_i-\langle\Delta p'_{\mathrm{ki}}\rangle_i+\langle\boldsymbol{n}_k\cdot\bar{\tau}_{kx}\rangle_i+\langle\boldsymbol{n}_{kw}\cdot\bar{\tau}_{kx}\rangle_w$$
$$-\frac{\partial}{\partial x}(\beta_k\rho_k\langle\delta v_{kx}\delta v_{kx}\rangle)-\frac{\partial}{\partial y}(\beta_k\rho_k\langle\delta v_{kx}\delta v_{ky}\rangle) \tag{19}$$

The momentum equation in the y direction is derived in a similar manner.

It is worthwhile to consider certain important differences between the current formulation and previous studies, such as that of Prosperetti and Jones.[9] Unlike previous studies, the interfacial pressure drag force per unit volume in the fifth term of the differential equation (19) and the apparent mass force per unit volume (10th term of the same equation) do not coincide. This flexibility allows modeling of these terms in various flow conditions independently. The second term on the right side of Eq. (15) considers the volume-averaged bulk pressure of phase k. With the term $(\partial/\partial x)\beta_k\langle p_k\rangle$ in the phase momentum equation, it leads to a fourth term of Eq. (19).

An explicit form of a term $\langle p_k\rangle(\partial\beta_k/\partial x)$ was removed from Eq. (19). Additionally, this term will be grouped with interfacial terms, such as $\Delta p_{\mathrm{ki}}(\partial\beta_k/\partial x)$, $\langle\Delta p'_{\mathrm{ki}}\rangle_i$, and $\langle\boldsymbol{n}_k\bar{\tau}_{kx}\rangle_i$, and a wall term $\langle\boldsymbol{n}_{kw}\bar{\tau}_{kx}\rangle_w$ under a surface integral and modeled as an algebraic source term in an upcoming momentum equation, Eq. (21). This is a different procedure from that documented by Prosperetti and Jones.[9] Our procedure distinguishes, separates, and groups the interfacial and the wall forces per unit volume, thereby yielding a useful insight into the mechanisms of interface pressure behavior and its modeling in various multiphase phenomena (separation behind the droplets, coalescence or breakup of droplets, etc.).

Also, unlike previous formulations (i.e., Ref. 9), the dispersed phase pressure in this paper is not necessarily equal to the bulk pressure of the continuous phase in the x-momentum equation (19). This aspect can be particularly significant if the dispersed phase moves through a continuous phase having rapid pressure variations, or different surrounding continuous phase(s), because there might be insufficient time for the dispersed phase pressure to adjust. An example is supercooled droplets distributed spatially throughout clouds at different elevations, with significantly different characteristics of the continuous (air) phase around the helicopter, that is, downwash and near-wall regions of a helicopter surface.

The interfacial pressure forces, such as drag force and apparent mass force per unit volume, would be difficult or inconvenient to evaluate in the current form of Eq. (19). For disperse phase motion, these forces are assumed to coincide. Therefore, they are grouped together. This result is the same as in Ref. 9. For our droplet flow model, these interfacial pressure forces are modeled with the interfacial and wall shear forces in an algebraic drag term. Unlike previous studies (i.e., Refs. 7, 11, and 16), the same difficulty of addressing the interfacial pressure forces is avoided. This is especially important in a spatially uniform dispersed flow.

In a dispersed-phase multiphase flow, the Reynolds stresses (analogous to the Reynolds stresses in a single-phase flow) arise from the dispersed-phase velocity deviations. These stresses represent information lost in the averaging procedure, which has to be supplied back. It is usually supplied by modeling. If a disperse phase is dilute, the disperse-phase Reynolds stresses arise as a result of the turbulence in the carrier phase. These stresses are modeled through the Boussinesq assumption. For example, for the y-convective term in the x-momentum equation, this yields

$$\beta_k \rho_k \langle \delta v_{kx} \delta v_{ky} \rangle = -\mu_k \left(\frac{\partial \langle v_{ky} \rangle}{\partial x} + \frac{\partial \langle v_{kx} \rangle}{\partial x} \right) \tag{20}$$

The Reynolds stresses in multiphase flows are more complicated to predict than the Reynolds turbulent stresses in a single-phase flow. In certain cases, the averaged Reynolds stresses in the dispersed phase can affect the motion of the carrier phase, particularly for dense multiphase flows.

Spatially averaged deviations in the transported quantities can occur from sources other than turbulence in the carrier phase. For example, the averaged Reynolds stresses can arise from different cross-phase transport processes at the interfacial boundary, such as processes involving variations of droplet velocities and sizes. Such processes can involve coalescence of different sized droplets with varying dynamic responses to averaged spatial variations in the surrounding continuous (air) phase, thereby affecting the local turbulence. These considerations can be particularly significant in confined regions of a flow, such as a contracted cross section, where smaller particles tend to follow the local velocity of the continuous phase, while larger particles tend to follow larger velocity scales. In these cases, the solution might require that the averaged equations of motion are solved for each group of droplets.

In Eq. (20), a droplet dynamic viscosity is used as an averaged assumption for the dispersed phase, as this viscosity is a property of the multiphase turbulent flow. Ideally, this viscosity depends on the detailed structure and length scales of the multiphase turbulent flow. Also, the Boussinesq assumption with volume-averaged velocities in Eq. (20) involves certain limitations. Although the droplet velocity deviations exist at the interfacial boundaries, they cannot be fully modeled in the uniform air flow because of limitations associated with a dilute flow model.

The detailed droplet behavior because of the presence of a wall, such as rotation or splash-back effects, is not fully modeled by the Boussinesq assumption. As droplets approach a surface, the volume-averaged collision effects (droplet/wall and droplet/droplet) become significant, in contrast to assuming droplet/air interactions to be dominant in a dilute flow formulation. In this near-wall case, modifications of the Reynolds stresses are needed. A Boussinesq assumption can be used with an effective viscosity, provided that the near-wall effects can be predicted through empiricism or other means. The presence of many droplets involves various length scales, droplet diameters, distances between droplets, and so on. Alternatively, the constitutive relations can be modeled by kinetic theory.

In a dilute flow with water droplets (dispersed phase) in air (continuous phase), a large density difference exists between the phases. As a result, the interfacial force of droplets acting to accelerate/decelerate the surrounding air phase is considered to be negligible. After substituting appropriate expressions for the interfacial drag forces (acting on the droplets from the carrier phase) and rearranging the pressure and stress terms, the spatially averaged x-momentum equation for the droplet phase is obtained as follows:

$$\begin{aligned} &\frac{\partial}{\partial t}(\beta\rho\langle v_x\rangle) + \frac{\partial}{\partial x}(\beta\rho\langle v_x\rangle\langle v_x\rangle) + \frac{\partial}{\partial y}(\beta\rho\langle v_x\rangle\langle v_y\rangle) \\ &\quad = \frac{1}{V}\sum v_x \dot{m} - \beta\frac{\partial}{\partial x}\langle p\rangle + \beta\frac{\partial}{\partial x}\langle \tau_{xx}\rangle + \beta\frac{\partial}{\partial y}\langle \tau_{yx}\rangle \\ &\quad - \frac{\partial}{\partial x}(\beta\rho\langle\delta v_x \delta v_x\rangle) - \frac{\partial}{\partial y}(\beta\rho\langle\delta v_x \delta v_y\rangle) - \beta_{V,x}(\langle v_x\rangle - \langle v_{x,a}\rangle) \end{aligned} \tag{21}$$

This equation governs the motion of the droplets. All symbols refer to the droplet phase, and the phase subscripts, such as d, are omitted, except v_a (air velocity). After including the gravity term, a similar equation is obtained in the y direction:

$$\begin{aligned} &\frac{\partial}{\partial t}(\beta\rho\langle v_y\rangle) + \frac{\partial}{\partial x}(\beta\rho\langle v_y\rangle\langle v_x\rangle) + \frac{\partial}{\partial y}(\beta\rho\langle v_y\rangle\langle v_y\rangle) \\ &\quad = \frac{1}{V}\sum v_y \dot{m} - \beta\frac{\partial}{\partial x}\langle p\rangle + \beta\frac{\partial}{\partial x}\langle \tau_{yx}\rangle + \beta\frac{\partial}{\partial y}\langle \tau_{yy}\rangle \\ &\quad - \frac{\partial}{\partial x}(\beta\rho\langle\delta v_y \delta v_x\rangle) - \frac{\partial}{\partial y}(\beta\rho\langle\delta v_y \delta v_y\rangle) \\ &\quad - \beta_{V,y}(\langle v_y\rangle - \langle v_{y,a}\rangle) + \beta\rho g_y \end{aligned} \tag{22}$$

Closure relations are required for computations based on Eqs. (21) and (22). For example, for Eq. (21) it yields

$$\beta_{V,x} = \frac{\beta f_x}{D_d^2 / 18\mu_d} \tag{23}$$

which holds for spherical droplets. The function f is the ratio of the drag coefficient to the Stokes drag (note: $f \to 1$ for Stokes drag). It is also called a drag factor. For the x direction, it is

$$f_x = (C_D/24) Re_{r,x} \tag{24}$$

Alternatively, expanding the representation for the Reynolds number,

$$f_x = (C_D/24)[D_d(|\langle v_x\rangle - \langle v_{x,a}\rangle|/\nu)] \tag{25}$$

Velocity differences between the droplets and air arise as a result of velocity gradients in the carrier phase, turbulent fluctuations of droplets and/or time-varying body forces on the droplets. For both low- and high-Reynolds-number flows, the drag factor f_x can be approximated by

$$f_x = 1 + 0.15 Re_{r,x}^{0.687} + 0.0175 Re_{r,x}\left(1 + 4.25.10^4 Re_{r,x}^{-1.16}\right)^{-1} \tag{26}$$

The drag factor f_y is obtained similarly as the drag factor f_x.

In this paper, the detailed volume averaging leading to Eqs. (21) and (22) was presented, so that the modeling assumptions and simplifications of other reduced models can be identified. For example, Ref. 23 includes only the drag-coefficient term of Eq. (22) and gravity force in the y direction,[23] and so a comparison with Eq. (22) identifies the assumptions made therein. In that case, the pressure interactions and spatial-averaging considerations are neglected. This approach assumes that the droplets are distributed uniformly within the averaging control volume. The gravity force on droplets is introduced in the droplet y-momentum equation. Furthermore, droplets are assumed to be spherically shaped and modeled numerically as solid particles. As a result, a potentially varying shape of droplets, together with a resulting momentum change as a result of this effect, is not modeled. Also, the cross-phase stresses are assumed to not affect the droplet shape. Evaporation, collisions, and coalescence of droplets are neglected, thereby neglecting cross-phase mass and momentum exchange as a result of these effects. Although this assumption is often adopted in the freestream region, its validity in the near-wall region is not well understood in the technical literature. In the near-wall region, the fraction of droplets in the multiphase averaging control volume changes appreciably.

D. Energy Equation

Pressure effects also appear in the energy equations, where heat-transfer processes are studied. Those volume-averaged equations have been derived previously by Naterer.[26] In this paper, the equations are summarized and focused specifically on the pressure effects. After substituting $\psi_k = e_k = \hat{e}_k + \frac{1}{2}\boldsymbol{v}_k \cdot \boldsymbol{v}_k$, $\boldsymbol{j}_k = \boldsymbol{q}_k + (p_k\bar{\bar{I}} - \bar{\bar{\tau}}_k) \cdot \boldsymbol{v}_k$, and $\hat{S}_k = \boldsymbol{F}_{b,k} \cdot \boldsymbol{v}_k + \hat{S}_{k,e}$ into Eq. (6), the one-dimensional form of the total energy equation becomes

$$\frac{\partial}{\partial t}(\beta_k\langle\rho_k e_k\rangle) + \frac{\partial}{\partial x}(\beta_k\langle\rho_k e_k u_k\rangle) + \frac{\partial}{\partial x}(\beta_k\langle q_{k,x}\rangle) + \frac{\partial}{\partial x}(\beta_k\langle p_k u_k\rangle)$$

$$- \frac{\partial}{\partial x}[\beta_k\langle \boldsymbol{n}_x(\bar{\bar{\tau}}_k \boldsymbol{v}_k)\rangle] + \frac{1}{V}\int_S [\dot{m}_k e_k + (\boldsymbol{q}_k + p_k \boldsymbol{v}_k - \bar{\bar{\tau}}_k \cdot \boldsymbol{v}_k) \cdot \boldsymbol{n}_k]\, \mathrm{d}S$$

$$= \beta_k\langle\rho_k(\boldsymbol{F}_{b,k} \cdot \boldsymbol{v}_k + \hat{S}_{k,e})\rangle \tag{27}$$

The symbol e_k represents the total energy, $\hat{e}_k$ is the internal energy, $\boldsymbol{q}_k$ is the Fourier heat flux (note that work done by pressure and viscous forces is shown within the same brackets), and the symbol $\hat{S}_k$ represents work caused by body forces. The physical processes on the surfaces are incorporated in the last term of Eq. (27). They represent the interfacial fluid–fluid and fluid–solid (wall) interactions.

In a uniformly dispersed droplet phase ($\beta = \text{constant}$), the pressure within a droplet (fourth term) can contribute to the net change of the total energy of the dispersed phase. This energy can involve compression of droplets by the carrier phase, and a contribution as a result of the change of kinetic energy of the droplet phase. If the pressure within the droplet phase is assumed to be constant, the work associated with the dispersed phase pressure is done only by the compression/expansion mode.

In a dilute flow of uniformly dispersed droplets, the change of total energy as a result of the dispersed-phase pressure is realized only from the change of kinetic energy of the droplet phase. In the sixth term in the Eq. (27), the total droplet energy can change as a result of the pressure work acting on the surface of this dispersed phase. If the droplet surface comes into close contact with a wall, the total energy of the droplet phase as a result of the pressure work contribution at the surface remains unchanged. Modeling of the bulk pressure contribution to the change of the total energy becomes significant if the droplets are transported from one region of multiphase flow to another region with a different carrier phase and bulk pressure. Such differences would require detailed modeling of the interfacial pressure effects, similarly to procedures outlined earlier for the momentum equations.

III. Conclusions

An Eulerian formulation is presented for multiphase turbulent flows with droplets. It considers the detailed volume averaging for the pressure and cross-phase momentum exchange processes in the dispersed (droplet) phase. Alternative formulations of the interfacial pressure drag and apparent mass forces are developed, thereby addressing shortcomings in previously derived modeling of the dispersed-phase pressure term $\langle p_k\rangle\nabla\beta_k$. Pressure differences across the interfacial boundary are considered, so that detailed modeling of such variations can be accommodated in the volume averaging. Moreover, additional interfacial and wall shear stresses are modeled as algebraic source terms in the equations of motion for the high- and low-Reynolds-number flows. Because of large density differences between the dispersed (droplet) and carrier (air) phases, the apparent mass force in the dilute droplet phase becomes negligible. Multiphase Reynolds stresses arise from turbulence in the carrier phase, as well as interactions with the dispersed phase. These turbulent stresses are modeled by the Boussinesq assmuption, which is considered to be a reasonable assumption for dilute flows. Also, the role of interfacial pressure terms in the energy equation is discussed. Based on these detailed treatments of various multiphase pressure effects, the limitations and approximations inherent in other simplified droplet flow models can be elucidated. Although such considerations have been previously documented for Lagrangian methods of tracking individual droplet trajectories, this paper outlines a new detailed treatment in an Eulerian framework with volume averaging.

Acknowledgments

Financial support from GKN Westland Helicopters, Ltd., and the Canada Foundation for Innovation, as well as advisory assistance of N. Popplewell (University of Manitoba), is gratefully acknowledged.

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